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Abstract

Analytical Treatment of Neutron Thermalization in Two Medium Wigner-Seitz Cells of Cylindrical Symmetry

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An analytical method is presented for solving the thermalization problem in space and energy for a cylindrical Wigner-Seitz cell, containing an infinitely long fuel rod surrounded by a moderator, which is treated as a monatomic gas. The energy loss of scattered neutrons in the fuel is neglected.

The Boltzmann equation is solved in both media by expanding the neutron distribution and the transfer scattering cross section in spherical harmonics. A system of coupled integro-differential equations results for the energy and space dependent expansion coefficients. This system first is solved for epithermal neutron energies ($\xi \geq$ 16 kT). In this region one can describe the energetical behaviour of the spectrum by a power series in $1/\sqrt{\xi}$ if one considers $1/\sqrt{\xi}$ -absorption only. In this region one cannot deal with the slowing down kernel for zero moderator temperature. Therefore all scattering integrals have been evaluated analytically with the asymptotic representation of the correct kernel.

This procedure leads to a system of inhomogeneous differential equations, which can be solved by the modified Besselfunctions of the second kind. The free constants in the fuel and moderator solutions are determined by the cell boundary conditions.

The epithermal solution is extrapolated to zero energies by multiplying with a "damping" function which vanishes with a sufficiently high power for $\mathcal{E} \to 0$. Then the difference between the true spectrum and this artificial function is expanded in a system of orthonormal functions with an exponential asymptotic behaviour. The result is an inhomogeneous equation for this difference spectrum. If one deals with the unsymmetrisized scattering kernel, the basis functions have to obey a certain condition, which guarantees the detailed balance of the representation of the kernel in this system. On the other hand the adjoint functions also have to include an exponential factor, because otherwise the expansion coefficients of the inhomogeneity may become infinity.

The general calculation to get the difference spectrum is straight forward. The homogeneous solution of the inhomogeneous equations in both the moderator and fuel media lead to an eigenvalue problem.

This method was first applied to the infinite medium case to look for convergence questions and consistency. As a test the integral neutron balance was calculated. The result is much better than one percent with four basis functions and an asymptotic expansion up to $1/\mathcal{E}^4$. The cell calculations, done in a consistent P_1-approximation, reproduce also the balance better than one percent with an epithermic expansion up to $1/\mathcal{E}^2$. The dependence of the spectra on the moderator mass is investigated, also the influence of anisotropic scattering. It is seen that the average fuel spectrum deviates appreciably from the fuel boundary spectrum.

The mathematical formulation is carried on to P_3-approximation, but numerical calculations have been done only in P_1-approximation so far.